Stoichiometries and Equilibrium Constants of Cyclodextrin-Surfactant Complexations

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General and rigorous theory for equilibrium concentrations of uncomplexed species and 1 : 1, 1 : 2, 2 : 1, and 2 : 2 complexes of cyclodextrin (D) and surfactant has been developed and applied to evaluate the binding constants of these complexations from surface tension data of aqueous solutions of dodecyl maltoside (DM) with α-, β-, and γ-D, reported by Saeger and Muller-Fahrnow. The orders of the binding constants for 1 : 1 and 2 : 1 complexations of D and DM are α-D > β-D > γ-D. Those for 1 : 2 and 2 : 2 complexations are γ-D > β-D > α-D. These orders are explained on the basis of the goodness of fit of the dodecyl chain into the cyclodextrin cavity. Furthermore, the theory for the effects of D on the molar conductivity and the critical micelle concentration (cmc) of an ionic surfactant is modified by accounting for the change of these values with the surfactant concentration and applied to evaluate the binding constants for 1 : 1, 1 : 2, and 2 : 1 complexations of sodium dodecyl sulfate (SDS) and β-D from conductance and cmc data reported by Palepu and Reinsborough. Although there are several uncertainties in experimental data and their interpretations for the β-D–SDS system, the 1 : 1 complexation is predominant and its binding constant appears to be 5000 $dm^3\ mol^{-1}$. Based on the present analysis, it is suggested that since a surfactant is a long and fine molecule, relative to aromatic compounds, its ternary and quaternary complexes with D's as well as its binary complex should be taken into consideration.

Cyclodextrins are a group of doughnut-shaped oligosaccharides built up from six, seven, or eight glucopyranose rings, respectively known as alpha, beta, and gamma cyclodextrin. The interior of the doughnut, lined with CH groups, provides a relatively hydrophobic environment. One side of the torus contains primary hydroxyl groups, whereas the secondary hydroxyl groups are located on the other side. This structure, shown in Fig. 1, allows the cyclodextrin cavity to entrap organic compounds (substituted aromatic compounds, amino acids, nucleotides, etc,), ions, and gases.[1,2] To a first approximation the magnitude of binding constants correlates with the fit of the guest in the cyclodextrin cavity. Substrate–cyclodextrin binding constants range from 10^2 to 10^7 $dm^3\ mol^{-1}$. The driving force for substrate–cyclodextrin complex formation has been discussed in terms of van der Waal's interactions, hydrogen bonding, hydrophobic interactions, and the release of high energy water molecules and of steric strains on complexation.[1,2]

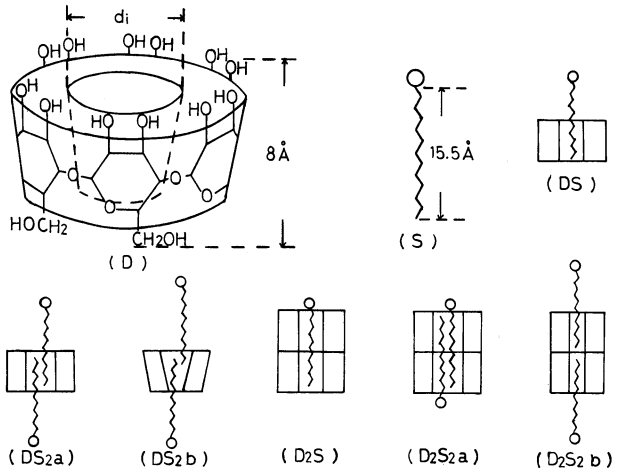

Fig. 1. Schematic structures of cyclodextrin (D), surfactant (S), and their complexes: for α-D, d_i=4.5—5.2 Å; for β-D, d_i=6.0—7.0 Å; for γ-D, d_i=7.5–8.5Å.[1,2]

Since cyclodextrins (D) provide chiral cavities, complex formation can be stereoselective and they can be used to separate enantiomers. Since they can accelerate chemical transfer reactions to a considerable extent, they have served as extremely useful models for enzyme-like interactions.[1] They can give rise to beneficial modifications of guest molecules not otherwise achievable; solubility enhancement, stabilization of labile guests, control of volatility and sublimation, and physical isolation of incompatible compounds. Since they are water-soluble and practically non-toxic, they are added into pharmaceuticals and foods, e.g., for stabilization of labile compounds and long-term protection of color, odor, and flavor.[2]

A number of studies are focussed on interactions of D's with ionic surfactants.[3-14] A 1 : 1 stoichiometric binding is presumed and its binding constant K_1 is evaluated.[3-12,14] The effects of the kind of D's and the head group and chain length of surfactants (S) on their complexations are also investigated.[4,8,10] However, the stoichiometry and magnitude of binding constants are still controversial. For instance, the binding constants of β-D with sodium dodecyl sulfate (SDS) are reported to be 300,[7] 356,[3] 1300—7230,[8] 3630,[6] 3200—18000,[14] and 25600 $dm^3\ mol^{-1}$,[10] and are dependent on the SDS concentration.[8,14] This wide spread in K_1 suggests that other stoichiometric bindings, such as D_2S[10,14] and D_2S_2,[10] occur.[8] To our knowledge, only a few reports deal with the interaction between a cyclodextrin and a nonionic surfactant.[13,15]

In this work we report a general quantitative treatment

for cyclodextrin–surfactant interactions including 1 : 1, 1 : 2, 2 : 1, and 2 : 2 complexes. Furthermore, we apply it to surface tension data for D's and dodecyl maltoside (DM)[15] and to the effects of β-D on the critical micelle concentration (cmc) and the molar conductivity of SDS,[8] since these experimental data are rather extensive.

Basic Theory

Material Balance for Cyclodextrin-Surfactant Systems. In general, we suppose four complex species of D and S, viz., DS, D_2S, DS_2, and D_2S_2:

$$\begin{array}{ccc} & & D_2S + S \\ & K_2 \nearrow & \searrow K_4 \\ 2D + 2S \overset{K_1}{\rightleftharpoons} & DS + D + S & D_2S_2. \\ & K_3 \searrow & \nearrow K_5 \\ & & DS_2 + D \end{array} \quad (1)$$

The equilibrium constants for these bindings are defined as

$$K_1 = [DS]/[D][S], \quad (2)$$

$$K_2 = [D_2S]/[DS][D], \quad (3)$$

$$K_3 = [DS_2]/[DS][S], \quad (4)$$

$$K_4 = [D_2S_2]/[D_2S][S], \quad (5)$$

and

$$K_5 = [D_2S_2]/[DS_2][D]. \quad (6)$$

These constants are connected by the equation of $K_2K_4=K_3K_5$. The total concentration C_D of D can be written as

$$\begin{aligned} C_D &= [D] + [DS] + 2[D_2S] + [DS_2] + 2[D_2S_2] \\ &= [D] + K_1[D][S] + 2K_1K_2[D]^2[S] + K_1K_3[D][S]^2 \\ &\quad + 2K_1K_3K_5[D]^2[S]^2. \end{aligned} \quad (7)$$

The total concentration C_S of S can be written as

$$\begin{aligned} C_S &= [S] + [DS] + [D_2S] + 2[DS_2] + 2[D_2S_2] \\ &= [S] + K_1[D][S] + K_1K_2[D]^2[S] + 2K_1K_3[D][S]^2 \\ &\quad + 2K_1K_3K_5[D]^2[S]^2. \end{aligned} \quad (8)$$

Three general cases including realistic ones will be considered; case i containing DS, D_2S, and DS_2, case ii containing DS, DS_2, and D_2S_2, and case iii containing DS, D_2S, and D_2S_2. For case i, combination of Eqs. 7 and 8 yields

$$\begin{aligned} &\{1 + 4K_1K_2C_D[S] - K_1(K_1 + 2K_3)[S]^2 - 4K_1^2K_3[S]^3 \\ &- 3K_1^2K_3^2[S]^4 - (1 - K_1[S] - 3K_1K_3[S]^2)[(1 + K_1[S] \\ &+ K_1K_3[S]^2)^2 + 8K_1K_2C_D[S]]^{1/2}\}/8K_1K_2[S] \\ &+ [S] - C_S = 0. \end{aligned} \quad (9)$$

When C_S, C_D, K_1, K_2, and K_3 are known, one can solve Eq. 9 by the Newton–Raphson method[16] to obtain free surfactant concentration [S]. Then one can obtain free cyclodextrin concentration [D] from

$$\begin{aligned} [D] = \{&-1 - K_1[S] - K_1K_3[S]^2 + [(1 + K_1[S] + K_1K_3[S]^2)^2 \\ &+ 8K_1K_2C_D[S]]^{1/2}\}/4K_1K_2[S]. \end{aligned} \quad (10)$$

Furthermore, the concentrations of DS, D_2S, and DS_2 can be calculated from Eqs. 2—4. Unless any D_2S is formed, viz., K_2=0, one can calculate [S] and [D] from

$$\begin{aligned} &K_1K_3[S]^3 + K_1[1 + K_3(2C_D - C_S)][S]^2 \\ &\quad + [1 + K_1(C_D - C_S)][S] - C_S = 0 \end{aligned} \quad (11)$$

and

$$[D] = C_D/(1 + K_1[S] + K_1K_3[S]^2). \quad (12)$$

When K_3=0, one can calculate [D] and [S] from

$$\begin{aligned} &K_1K_2[D]^3 + K_1[1 + K_2(2C_S - C_D)][D]^2 \\ &\quad + [1 + K_1(C_S - C_D)][D] - C_D = 0 \end{aligned} \quad (13)$$

and

$$[S] = C_S/(1 + K_1[D] + K_1K_2[D]^2). \quad (14)$$

For case ii where three complexes of DS, DS_2, and D_2S_2 are present, one can obtain [S] from

$$\begin{aligned} &(1 - K_1K_3[S]^2)\{[1 + K_1[S] + K_1K_3[S]^2 - [(1 + K_1[S] \\ &+ K_1K_3[S]^2)^2 + 8K_1K_3K_5C_D[S]^2]^{1/2}\}/4K_1K_3K_5[S]^2 \\ &+ [S] - C_S + C_D = 0. \end{aligned} \quad (15)$$

For case iii where DS, D_2S, and D_2S_2 are present, one can obtain [S] from

$$\begin{aligned} &\{(1 + K_1[S])(1 - K_1[S] + 2K_4[S]) - 4K_1K_2[S]C_D(1 + K_4[S]) \\ &+ (2K_1K_4[S]^2 + K_1[S] - 1)[(1 + K_1[S])^2 + 8K_1K_2[S]C_D \\ &(1 + K_4[S])]^{1/2}\}/8K_1K_2[S](1 + K_4[S])^2 + [S] - C_S + C_D \\ &= 0. \end{aligned} \quad (16)$$

Unless micelles contain any form of D, one can set [S] to cmc_0, viz., the cmc in the absence of D. By using the above equations for C_S, one can calculate cmc as a function of C_D in the presence of D. For instance, the cmc of a nonionic surfactant, under the condition where DS only is present as a complex, is given by[3]

$$\mathrm{cmc} = K_1\mathrm{cmc}_0 C_D/(1 + K_1\mathrm{cmc}_0) + \mathrm{cmc}_0. \quad (17)$$

When DS and DS_2 only are present as complexes, one can write cmc as

$$\mathrm{cmc} = (K_1\mathrm{cmc}_0 + 2K_1K_3\mathrm{cmc}_0^2)C_D/ (1 + K_1\mathrm{cmc}_0 + K_1K_3\mathrm{cmc}_0^2) + \mathrm{cmc}_0. \quad (18)$$

Here it is noted that both Eqs. 17 and 18 predict linear increases of cmc with increasing C_D. This slope should be less than unity for Eq. 17, whereas it can be greater than unity for Eq. 18 when $K_1K_3\mathrm{cmc}_0^2$ is greater than unity.

Ionic Surfactants. Unless ion-pairing occurs for any complex, Eqs. 2—16 may also hold for the complexation of D and uni-univalent ionic surfactant (SNa). However, expressions for cmc may be modified, since cmc decreases with increasing counterion concentration $[Na^+]$ as[17]

$$\mathrm{cmc} = \mathrm{cmc}_0^{1+\beta}[Na^+]^{-\beta} = a[Na^+]^{-\beta}. \quad (19)$$

Here $a = \mathrm{cmc}_0^{1+\beta}$ and β is an empirical constant obtained from the dependence of cmc on the concentration of salt added. In the presence of SNa and D, one can write [S] as

$$[S] = a\mathrm{cmc}^{-\beta}. \quad (20)$$

When DS and D_2S are present, by using Eqs. 13, 14, and 20, one can calculate cmc (C_S) from

$$8K_1K_2a\mathrm{cmc}^{1-\beta} - K_1(8K_2 - K_1)a^2\mathrm{cmc}^{-2\beta} - 4K_1K_2aC_D\mathrm{cmc}^{-\beta} - 1 - (K_1a\mathrm{cmc}^{-\beta} - 1)[(1 + K_1a\mathrm{cmc}^{-\beta})^2 + 8K_1K_2aC_D\mathrm{cmc}^{-\beta}]^{1/2} = 0. \quad (21)$$

When DS only is present, by setting $K_2 = 0$ in Eq. 21, one can calculate cmc from

$$\mathrm{cmc} = K_1a\mathrm{cmc}^{-\beta}C_D/(1 + K_1a\mathrm{cmc}^{-\beta}) + a\mathrm{cmc}^{-\beta}. \quad (22)$$

When β is zero in this equation, Eq. 22 reduces to Eq. 17. When DS and DS_2 are present as complexes, by using Eq. 11, one can calculate cmc (C_S) from

$$K_1K_3a^3\mathrm{cmc}^{-3\beta} + K_1a^2[1 + K_3(2C_D - \mathrm{cmc})]\mathrm{cmc}^{-2\beta} + a[1 + K_1(C_D - \mathrm{cmc})]\mathrm{cmc}^{-\beta} - \mathrm{cmc} = 0. \quad (23)$$

When DS and D_2S are present as complexes, the apparent molar conductivity Λ of the surfactant below the cmc may be written approximately as

$$\Lambda = (\lambda_S[S] + \lambda_{DS}[DS] + \lambda_{D_2S}[D_2S])/C_S + \lambda_{Na^+}. \quad (24)$$

Here λ_i denotes the molar conductivity of ionic species i. The decrease in molar conductivity $\Delta\Lambda$ upon addition of D may be written as

$$\Delta\Lambda = (\Delta\lambda_1[DS] + \Delta\lambda_2[D_2S])/C_S. \quad (25)$$

Here $\Delta\lambda_1 = \lambda_S - \lambda_{DS}$ and $\Delta\lambda_2 = \lambda_S - \lambda_{D_2S}$. Since the order of molecular sizes of S, DS, and D_2S is $D_2S > DS > S$, it can be predicted that the order of these molar conductivities is $\lambda_S > \lambda_{DS} > \lambda_{D_2S}$. When we attempt to apply Eq. 25 to conductance data, four parameters of K_1, K_2, $\Delta\lambda_1$, and $\Delta\lambda_2$ must be estimated. When DS only is present as a complex, Eq. 25 reduces to[4]

$$\Delta\Lambda = \Delta\lambda_1\{K_1(C_S + C_D) + 1 - [(K_1(C_S + C_D) + 1)^2 - 4K_1^2C_SC_D]^{1/2}\}/2K_1C_S. \quad (26)$$

When DS and DS_2 are present as complexes, $\Delta\Lambda$ may be written as

$$\Delta\Lambda = (\Delta\lambda_1[DS] + \Delta\lambda_3[DS_2])/C_S. \quad (27)$$

Here $\Delta\lambda_3 = 2\lambda_S - \lambda_{DS_2}$. When we attempt to apply Eq. 27 to conductance data, four parameters of K_1, K_3, $\Delta\lambda_1$, and $\Delta\lambda_3$ must be estimated.

Results

Surface Tension Data for the D-DM Systems. Surface tensions of aqueous DM solution containing either α-, β-, or γ-D were measured at 20 °C by Saeger and Muller-Fahrnow.[15] They suggested that the complex formation for these systems is more complicated than a simple 1 : 1 stoichiometry.

From the Gibbs adsorption equation,[18] the decrease in surface tension σ for the present conditions may generally be written as

$$-\mathrm{d}\,\sigma = RT(\Gamma_S\mathrm{dln}[S] + \Gamma_D\mathrm{dln}[D] + \Gamma_{DS}\mathrm{dln}[DS] + \Gamma_{D_2S}\mathrm{dln}[D_2S] + \Gamma_{DS_2}\mathrm{dln}[DS_2] + \Gamma_{D_2S_2}\mathrm{dln}[D_2S_2]), \quad (28)$$

where Γ_i denotes the surface excess of species i at the air/water interface. Since the addition of α-, β-, and γ-D does not decrease the surface tension of water, we can safely set Γ_D to zero; viz., the cyclodextrins are surface-inactive.[15] Therefore the complexes of DS, D_2S, DS_2, and D_2S_2 are expected to be less surface-active than DM. When competitive adsorption of these complexes and DM takes place on the aqueous surface, we may assume that the surfactant only is predominantly adsorbed, regardless of the presence of the complexes. This assumption is supported by the experimental results shown in Figs. 2—4. The solid lines show the observed σ values, which were read from Fig. 1 of Ref. 15 after its expansion. Here it is noted that the surface tension above the cmc is independent of C_D. Therefore we can assume that the surface tension shown in these figures depends on free DM concentration [S] only.

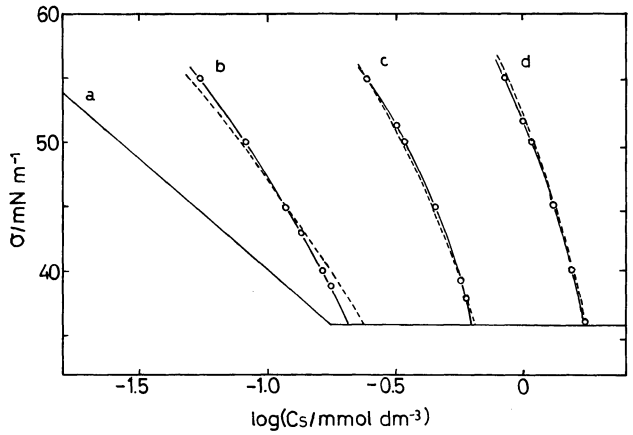

Fig. 2. Dependence of surface tension on the DM concentration in the presence of α-D at four concentrations (mmol dm^{-3}): a, 0; b, 0.1; c, 0.6; d, 2.1. The solid lines and hollow circles show the observed values[15] and the dashed lines are calculated on the basis of model α2.

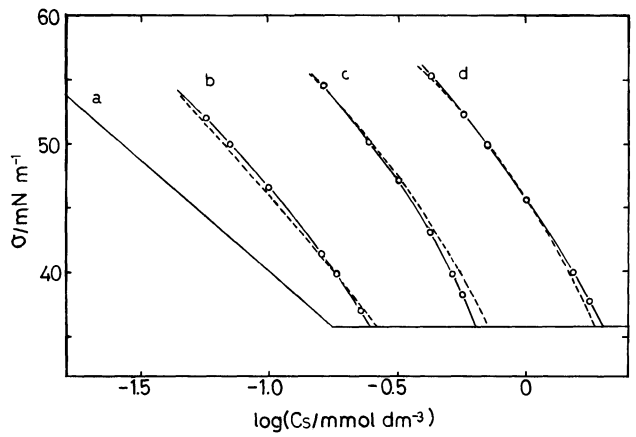

Fig. 3. Dependence of surface tension on the DM concentration in the presence of β-D at four concentrations (mmol dm^{-3}): a, 0; b, 0.1; c, 0.57; d, 1.74. The solid lines and hollow circles show the observed values[15] and the dashed lines are calculated on the basis of model β2.

The concentration dependence of σ for DM solutions may be written between 36 and 56 mN m^{-1} as[15]

$$\sigma = -17.69 \log [S] + 22.22. \quad (29)$$

From Eqs. 28 and 29 we can estimate Γ_S to be 3.16×10^{-10} mol cm^{-2} in the absence of D. This adsorption amount corresponds to a molecular occupied area of 52.6 Å^2. The estimation of binding constants was carried out by minimizing the SS value defined by

$$SS = \sum_{i=1}^{18} (\sigma_{i,obsd} - \sigma_{i,calcd})^2 . \quad (30)$$

The standard deviation in the estimated value of an adjustable parameter was also calculated[19] and will be shown in tables. Six observed σ values for each D concentration are shown by the open circles in Figs. 2—4. For nonlinear least-squares fitting,[19] 18 data points were used for each system of D and DM.

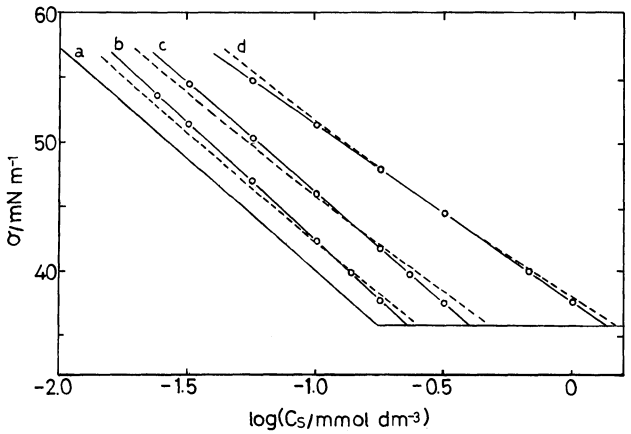

Fig. 4 Dependence of surface tension on the DM concentration in the presence of γ-D at four concentrations (mmol dm^{-3}): a, 0; b, 0.6; c, 2.0; d, 6.63. The solid lines and hollow circles show the observed values[15] and the dashed lines are calculated on the basis of model γ4.

As a first model for α-D, we considered the case where DS complex only is formed; $K_2=0$ and $K_3=0$. If an initial value of K_1 is given, we can calculate [S] for a set of C_S and C_D from Eq. 11 with $K_3=0$. The σ value corresponding to this [S] value can be calculated from Eq. 29. Then the SS value for 18 sets of C_S and C_D is calculated from Eq. 30. This procedure is repeated by adjusting the K_1 value, until the SS value is minimized. Thus we obtained best fit values of $K_1=35200$ dm^{3}mol^{-1} and SS=42.49mN^2 m^{-2}. These values are shown as model α1 in Table 1. In the case where complexes, DS and D_2S, only are present (model α2), we used Eqs. 13 and 14 to calculate [S]. This model is better fit than model α1, as shown in Table 1. The concentration dependence of σ thus calculated is shown by the dashed lines in Fig. 2. Model α3, in which the complexes of DS and DS_2 are present, is better fit than model α1, but the best fit value of K_3 is negative. Therefore, this is an unrealistic model. For model α4 which accounts for complexes of DS, D_2S, and DS_2, we used Eq. 9. For model α5, we used Eq. 16. This is also an unrealistic model, since the best fit K_4 value is negative.

To explain the σ data on β-D shown in Fig. 3, we considered four models. As Table 1 shows, model β2, presuming two complexes of DS and DS_2, is the best of them. The theoretical σ value based on this model is shown in Fig. 3. For γ-D, model γ4 taking into account three complexes of DS, DS_2, and D_2S_2 is the best in Table 1. It is remarkable that the second binding (K_3) of DM is easier than the first binding (K_1). As Fig. 4 shows, the experimental feature for γ-D is quite different from those

Table 1. Best Fit Values of Binding Constants for Dodecyl Maltoside and Cyclodextrins, Estimated from Surface Tension Data[a)]

Model	Eq.	Binding constants/dm^3 mol^{-1}					SS
		K_1	K_2	K_3	K_4	K_5	mN^2 m^{-2}
				α-Cyclodextrin			
α1	11	35200±2400	—	—	—	—	42.49
α2	13, 14	38400±1100	1150±190	—	—	—	5.99
α3	11	40800±1800	—	−670±40	—	—	15.25
α4	9	38400±1500	1160±270	3±155	—	—	6.01
α5	16	38400±200	1060±710	—	−40±590	—	4.31
				β-Cyclodextrin			
β1	11	26200±1900	—	—	—	—	42.29
β2	11	21500±1000	—	1380±260	—	—	6.47
β3	9	22300±1600	−68±100	1170±420	—	—	6.32
β4	15	21300±300	—	1420±130	—	23±330	6.49
				γ-Cyclodextrin			
γ1	11	682±40	—	—	—	—	25.30
γ2	11	470±40	—	3640±940	—	—	6.07
γ3	9	431±9	−5±10	4600±180	—	—	7.32
γ4	15	407±10	—	3080±10	—	212±10	4.66

a) Surface tension data are taken from Ref. 15.

Table 2. Best Fit Values of Binding Constants for Sodium Dodecyl Sulfate and β-Cyclodextrin, Estimated from Conductance Data[a)]

Model	Eq.	Binding constants/dm^3 mol^{-1}			Conductance differences/S cm^2 mol^{-1}			SS
		K_1	K_2	K_3	$\Delta\lambda_1$	$\Delta\lambda_2$	$\Delta\lambda_3$	S^2 cm^4 mol^{-2}
1	11, 26	5000±800	—	—	11.1±0.1	—	—	2.07
2	13, 25	5500±1900	230±230	—	11.2±0.6	10.7±0.5	—	1.92
3	11, 27	5400±1000	—	17±101	11.1±0.2	—	2.6±45.6	2.01

a) Conductance data are taken from Ref. 8.

for α- and β-D. The theoretical line based on model γ4 is well fit to the experimental results.

The σ vs. C_S plot for a surfactant breaks at the cmc. As Figs. 2—4 show, there is good agreement between the theoretical and experimental cmc values for the three systems.

Conductance Data for the β-D-SDS System. Palepu and Reinsborough reported detailed conductance data on the β-D-SDS system and obtained, by fitting Eq. 25 to these data, the following values of $\Delta\lambda_1$ (S cm^2 mol^{-1}) and K_1 (dm^3 mol^{-1}) at a constant C_S (mmol dm^{-3}): $\Delta\lambda_1$=10.5 and K_1=7230 at C_S=1, $\Delta\lambda_1$=11.6 and K_1=4690 at C_S=2, $\Delta\lambda_1$=11.3 and K_1=3340 at C_S=3, and $\Delta\lambda_1$=12.5 and K_1=1380 at C_S=5.[8)] Using these values, we calculated six "observed" $\Delta\Lambda$ values for each SDS concentration from Eq. 25. These "observed" values are shown at four SDS concentrations in Fig. 5. Since the K_1 values obtained by Palepu and Reinsborough depend on C_S, we attempted to reinterpret their conductance data (shown in Fig. 5) on the basis of a few models.

First, we used Eq. 26 under the assumption that K_1 and $\Delta\lambda_1$ both are independent of C_S and C_D, and obtained best fit values of K_1 and $\Delta\lambda_1$ by using the 24 data points shown in Fig. 5. This is model 1 shown in Table 2. The

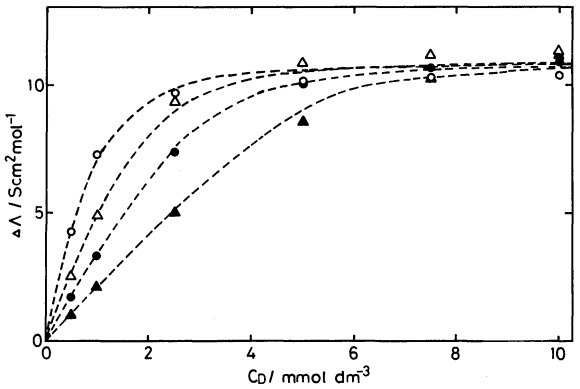

Fig. 5. Decreases in molar conductivity of SDS with addition of β-D at four SDS concentrations (mmol dm^{-3}): ○, 1; △, 2; ●, 3; ▲, 5. These data are taken from Ref. 8. The dashed lines are calculated on the basis of model 1 shown in Table 2.

theoretical values based on this model are shown by the dashed lines in Fig. 5. In model 2, we used Eqs. 13 and 25 under the assumption that K_1, K_2, $\Delta\lambda_1$, and $\Delta\lambda_2$ are independent of C_S and C_D. A wide range of these values was attempted as initial values for fitting and best fit

values are shown in Table 2. In addition, unexpectedly, the $\Delta\lambda_1$ value is larger than the $\Delta\lambda_2$ value. In model 3, two complexes of DS and DS_2 are assumed to be present. The agreement between this model and experiment is close to those for models 1 and 2. The estimated values of K_2 and K_3 are much smaller than those of K_1, suggesting that the 1 : 1 complex predominates over the 1 : 2 and 2 : 1 complexes for the β-D–SDS system.

Cmc Data for the β-D–SDS System. The effect of β-D on the cmc of SDS was investigated by Palepu and Reinsborough.[8] Their cmc values are shown as a function of C_D in Fig. 6.

Table 3 shows the best fit parameters of four models to the observed cmc values. A β value of 0.679 is employed.[19] Although model 4 (Eq. 17) was employed in the literature,[3,7] this is the worst model in Table 3. The K_1 value of 210 $dm^3\ mol^{-1}$ is comparable with the literature values[3,7] obtained from the same equation. These values, however, are smaller by one or two orders than those obtained by conductance[6,8,14] and fluorescence probe[10] methods. From Eq. 22, which takes into account the total counterion concentration, we can obtain a reasonable value of K_1=1210 $dm^3\ mol^{-1}$.

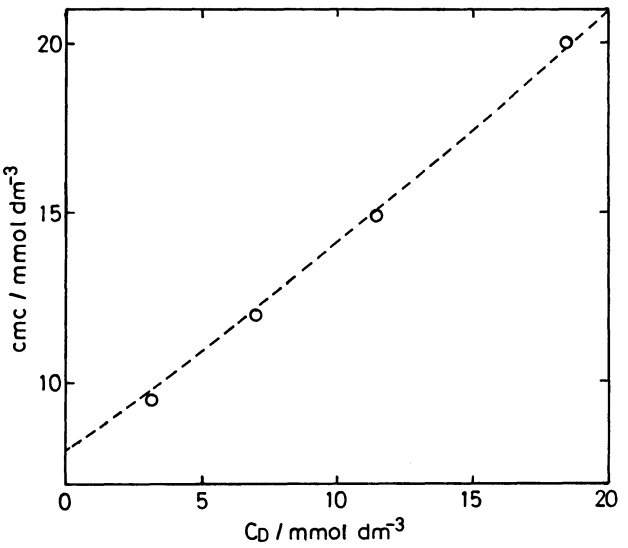

Fig. 6. Changes of cmc of SDS with the concentration of β-D at 25°C. The experimental values (o) are taken from Ref. 8 and the dashed line is calculated on the basis of model 5.

When K_2 or K_3 is taken into consideration, the agreement between theory and experiment is further improved, but negative values of K_2 and K_3 are obtained. When the experimental errors in cmc are taken into consideration, the difference in SS among the four models shown in Table 3 is small. For instance, as Fig. 6 shows, the difference between theory (model 5) and experiment is minor. If a cmc_0 value of 7.5 mmol dm^{-3} for SDS, obtained from linear extrapolation of the cmc values (Fig. 6) to C_D=0,[8] was employed, smaller SS values and more reasonable binding constants are obtained. Therefore, the cmc data seem to be insufficient to test the validity of the models.

Discussion

The D–DM Systems. General and rigorous theory accounting for complexes of DS, D_2S, DS_2, and D_2S_2 has been developed and its usefulness has been demonstrated by using some surface tension, conductance, and cmc data. To our knowledge, this is a first systematic paper which reports equilibrium constants for complexation of cyclodextrins with a nonionic surfactant.

As Table 1 shows, the agreement between theory and experiment is much improved by considering ternary complexes (D_2S and DS_2) and a quaternary complex (D_2S_2). The relative magnitude of the binding constants for the three cyclodextrins is explicable in terms of their cavity size. The order of the K_1 value is α- > β- > γ-D. A possible explanation for this order is the difference in the cavity size of the host molecules:[10] The dodecyl chain of DM fits snugly into α-D but rattles around inside the γ-D catity. The order of binding constants of the 1 : 1 complexes of sodium perfluorooctanoate with D's is β- > γ- > α-D.[11] This order is explicable by taking into account that the perfluorocarbon chain is larger than the hydrocarbon chain and the α-D cavity. A DM molecule can form a D_2S complex with two α-D molecules (model α2 in Table 1). This is consistent with the observations that the cyclodextrin/fatty acid ratio in complexes increases with increasing number of carbon atoms in the fatty acid[21] and that SDS forms a D_2S complex with α-D.[13] As the K_3 values show, the order of binding constants of the DS_2 complexes is γ- > β- > α-D. This is also expected from the cavity size. From the complexes built up by the Corey–Pauling–Koltun

Table 3. Best Fit Values of Binding Constants for Sodium Dodecyl Sulfate and β-Cyclodextrin, Estimated from Cmc Data[a]

Model	Eq.	β	Binding constants/$dm^3\ mol^{-1}$			SS
			K_1	K_2	K_3	$mmol^2\ dm^{-6}$
4	17	0	210±18	—	—	0.674
5	22	0.679	1210±130	—	—	0.201
6	21	0.679	733±80	−29±3	—	0.033
7	23	0.679	3600±1000	—	−19±3	0.015

a) Cmc data are taken from Ref. 8 and a cmc value of 8.0 mmol dm^{-3} for SDS is employed.

molecular model, the structure of the γ-D complex is expected to be DS_2a in Fig. 1 and that of the β-D complex to be DS_2b. γ-Cyclodextrin appears to form the D_2S_2 complex, which has the structure of D_2S_2a shown in Fig. 1. If α-D forms such a quaternary complex, its structure may be written as D_2S_2b. γ-Cyclodextrin and pyrene form a 2:2 complex,[22] which may have a structure similar to D_2S_2b.

The β-D-SDS System. As was described in the introduction, the K_1 values for β-D–SDS complexation range from 300 to 25600 $dm^3\ mol^{-1}$.[7,10] Furthermore, the K_1 value, as estimated from Eq. 26, decreases with increasing SDS concentration,[8,14] although the absolute values of K_1 obtained by Palepu and Reinsborough[8] are about halves of those by Aman and Serve.[14] If the conductance data by the latter group was employed, we would obtain about two times the K_1 values shown in Table 2. The apparent dependence of K_1 on C_S may come from the assumption that the conductance difference $\Delta\lambda_1$ is dependent on the SDS concentration. The $\Delta\lambda_1$ value for the β-D–SDS system increases with an increase in C_S, but that for the β-D–tetradecyltrimethylammonium bromide system decreases with an increase in C_S. Therefore, our assumption that the conductance differences are independent of C_S and C_D will be a good approximation at the present time. It is not easy to determine the values of K_2 and K_3 from conductance data, when these values are much smaller than K_1. The molar conductivity of SDS at 1 mmol dm^{-3} decreases with addition of α-D up to 3 mmol dm^{-3} and slightly increases with further addition of α-D.[13] It is not easy to interpret this result. From conductance data on mixtures of sodium alkanesulfonates with α-D at 25 °C, reported by Satake et al,[4] we can estimate the values of $\lambda_{DS}+\lambda_{Na^+}$. Regardless of the number of carbon atoms in the surfactants, these values are about 57.5 S $cm^2\ mol^{-1}$. This independence of λ_{DS} from the alkyl chain length suggests that the translational mobility of DS ion is controlled mainly by the size of the cyclodextrin, the bigger component. The same tendency is found in the sodium alkanesulfonates–β-D-systems;[6] the values of $\lambda_{DS}+\lambda_{Na^+}$ are about 56.5 S $cm^2\ mol^{-1}$, regardless of the alkyl chain length. For these surfactants, as expected, the molar conductivity decreases with increasing chain length of the surfactant.[4,6] However, the order of the molar conductivity for sodium alkanoates is octanoate > heptanoate > hexanoate > dodecanoate. The order of the $\Delta\lambda_1$ value is hexanoate > heptanoate > octanoate > decanoate > nonanoate > dodecanoate.[9] For mixtures of SDS with cyclodextrins, the order of the $\Delta\lambda_1$ value is α-D > β-D > γ-D[8] and for mixtures of sodium perfluorooctanoate with cyclodextrins, the order is β-D > γ-D > α-D.[11] These data reported by Palepu et al. demonstrate that we have serious difficulties in predicting the $\Delta\lambda_1$ value from only a knowledge of complex species.

Applying Eq. 17 to the cmc data determined by Palepu and Reinsborough,[8] we obtained a K_1 value of 210 $dm^3\ mol^{-1}$. This value is smaller than 300[7] and 356[3] obtained by the same method. Since we used the cmc values obtained by Palepu and Reinsborough,[8] the K_1 values (Table 3) calculated from Eqs. 21—23 may be too small.

The binding constants, K_1 and K_2, for D and long chain ionic surfactants increase with increasing number of carbon atoms of the surfactants, although short chain ionic surfactants form only a 1:1 complex.[4,8–10,13] This result shows that hydrophobic interactions play a predominant role in the complexation. Our K_1 value for β-D and DM is close to 25600 $dm^3\ mol^{-1}$, which was estimated by taking into account 1:1 and 2:1 complexations of β-D and SDS.[10] However, β-D and DM form a 1:2 complex but do not form any 2:1 complex, as shown in Table 1. The reason for this discrepancy between SDS and DM is unknown. Several ions, such as I^-, Br^-, Cl^-, and NO_3^- can be entrapped into β-D.[23] This result shows a complicated nature of the cavity. Some Na^+ ions might be associated with the 1:1 complex.[11] If this is true, Eqs. 24—27 must be modified.

In conclusion, since aliphatic compounds including surfactants are long and fine molecules, relative to aromatic compounds, their ternary and quaternary complexes with cyclodextrins as well as their binary complex should be taken into account. The surface tension method can be used as a unique tool for the investigation of cyclodextrin–surfactant interactions. Although there are several uncertainties in cmc and conductance data and their interpretations for the β-D–SDS system, DS is the predominant complex and the K_1 value appears to be 5000 $dm^3\ mol^{-1}$. The present results provide useful data for industrial applications of cyclodextrins and for understanding the behavior of cyclodextrins in bodies.

References

1) M. L. Bender and M. Komiyama, "Cyclodextrin Chemistry," Springer-Verlag, Berlin (1978).
2) W. Saeger, *Angew. Chem., Int. Ed. Engl.*, **19**, 344 (1980).
3) T. Okubo, H. Kitano, and N. Ise, *J. Phys. Chem.*, **80**, 2661 (1976).
4) I. Satake, T. Ikenoue, T. Takeshita, K. Hayakawa, and T. Maeda, *Bull. Chem. Soc. Jpn.*, **58**, 2746 (1985).
5) S. Hashimoto and J. K. Thomas, *J. Am. Chem. Soc.*, **107**, 4655 (1985).
6) I. Satake, S. Yoshida, K. Hayakawa, T. Maeda, and Y. Kusumoto, *Bull. Chem. Soc. Jpn.*, **59**, 3991 (1986).
7) J. Geoger and S. Desmettre, *J. Colloid Interface Sci.*, **118**, 192 (1987).
8) R. Palepu and V. C. Reinsborough, *Can. J. Chem.*, **66**, 325 (1988).
9) R. Palepu, J. M. Richardson, and V. C. Reinsborough, *Langmuir*, **5**, 218 (1989).
10) J. W. Park and H. J. Song, *J. Phys. Chem.*, **93**, 6454 (1989).

11) R. Palepu and V. C. Reinsborough, *Can. J. Chem.*, **67**, 1550 (1989).
12) C. D. Lavandier, M. P. Pelletier, and V. C. Reinsborough, *Aust. J. Chem.*, **44**, 457 (1991).
13) A. Hersey, B. H. Robinson, and H. Kelley, *J. Chem. Soc., Faraday Trans. 1*, **82**, 1271 (1986).
14) E. S. Aman and D. Serve, *J. Colloid Interface Sci.*, **138**, 365 (1990).
15) W. Saeger and A. Muller-Fahrnow, *Angew. Chem., Int. Ed. Engl.*, **27**, 393 (1988).
16) R. G. Mortimer, "Mathematics for Physical Chemistry," Macmillan, New York (1981), p. 249.
17) M. L. Corrin and W. D. Harkins, *J. Am. Chem. Soc.*, **67**, 682 (1947).
18) A. W. Adamson, "Physical Chemistry of Surfaces," Interscience, New York (1960), Chap. 2.
19) K. Yamaoka, "Yakubutsu Tainai Dotai Kaiseki Ho," Nankodo, Tokyo (1984).
20) R. J. Williams, J. N. Phillips, and K. J. Mysels, *Trans. Faraday Soc.*, **51**, 728 (1955).
21) H. Schlenk and D. M. Sand, *J. Am. Chem. Soc.*, **83**, 2312 (1961).
22) T. Yorozu, M. Hoshino, and M. Imamura, *J. Phys. Chem.*, **86**, 4426 (1982).
23) R. P. Rohrbach, L. J. Rodriguez, E. M. Eyring, and J. F. Wojcik, *J. Phys. Chem.*, **81**, 944 (1977).